Fourier Analysis and Change Detection

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Dynamic Network Analysis

- Key focus – Networks change over time
- Summary statistics – typically average all data
  - Useless for seeing changes over time
- Longitudinal Networks and Change
  - Getting longitudinal networks from communications logs
  - Stability, Evolution, Shock, Mutation
- Statistical Models of Networks to Detect Change
  - Link Probability Model (LPM) for Stability
  - Actor-Oriented Models for Evolution
  - Multi-Agent Simulation for Evolution, Shock, and Mutation
- Network Change Detection Algorithms
- Fourier Analysis to remove periodic variations
Basic Issue

• Real Social Networks are not time independent
• Over time the set of nodes change
  – Agents die, agents are born
  – If data set has limited geographic focus,
    • Agents can enter region under study
    • Agents can leave region under study
• Network connections between agents can change
  – A network link between two agents can disappear
    • Two family members have a fight and refuse to talk to each other
  – A new network link can be created
    • People meet new people and form new relationships
    • Advertising campaigns can convince people to follow companies

Types of Changes in Network Data

• Stability: Relationships remain statistically the same over time
  – If you are a signal processing person, the Network is "Ergodic"
• Evolution: Interaction among agents cause the relationships to change over time.
  – All link weights / costs are evolving over time during observations
• Shock: Change is exogenous to the social group.
  – E.g., like an earthquake hits Southern California
• Mutation: A shock stimulates evolutionary behavior.
  – E.g., after earthquake, people form many new links trying to survive
Dynamic Metrics on Over-Time Data

- Identifying central nodes in a network

Dynamic Changing Network Structure!!!

Proxy for Network Data

- Ideal approach – directly sample network each time period
  - E.g., have every member of society fill out survey every time period
  - Limited to very small societies
- Or, tracking changes over time using communications data
  - Communication is “proxy” for a network tie
  - Taking large amounts of communication data gives an approximate picture of the underlying social network (with some concerns)
  - Can use it to find Key Agents and other Social Structure measures
- Communication log data available from many sources
  - Cell Phone Service Providers – call logs, txt logs
  - E-mail Data logs – available within organization
  - Twitter, Facebook, FourSquare, etc.
  - Building Sensors, Cell Phone Sensors, RFID Tags, etc.
Example: Temporal Social Network

- ACM Hypertext 2009 Conference
  - Badges with RFIDs
- Close Range Face-to-Face Contact
  - 1 - 1.5 meters of one another
  - Human body acts as an RF shield
- Collect sensor data every 20 seconds for 2.5 days
  - 20,818 real time data updates
  - 113 participants, 2196 undirected, weighted links

Socio-Patterns: Betweenness Centrality Distribution

44/113 have 0 betweenness at end of Conference
Are they all the same?
Socio-Patterns: Betweenness Over-Time Trends

Changes in Network Data

• Various measures of a network are taken for a window at each time point.
• Change detection: quickly determine that a change occurs.
• Change point identification: when did the change occur.
Change Detection

• Goal: Rapidly detect that a change has occurred
• Detect shocks, not evolutionary changes
  – Evolutionary change: change due to interaction among actors in a network
    • Example: change of interaction patterns over time among new students as they get to know each other
  – Shock: change reason is exogenous to the network
    • Example: change of interaction patterns among students after they graduate
  – Another way to say it: detect “fast” change not “slow” change
• Another goal is to identify change point
  – Likely time when change occurred
  – Limits the scope of explanation for network change

Statistical Process Control (SPC)

• Change detection based on SPC
• Statistical Process Control
  – Used in manufacturing to maintain quality control
  – Monitors a process to detect potential changes
  – Calculates a statistic from observed measurements of a process and compares it to a decision interval
  – If the statistic exceeds the decision interval, it is said to “signal”, that a potential change may have occurred
  – A quality engineer will then begin to search for the specific cause of change
Statistical Models of Networks

**Link Probability Model (LPM) for Stability**

- LPM is a model for a network in *Stability*
- The probability that an email is sent from *i* to *j* within some period of time *t* is:
  \[ p = \int_0^t f_{ij}(x | \theta_{ij}) \, dx \]
  - (\( p \) as a function of \( t \) is a CDF; \( f \) is the PDF that best fits cell \( ij \) in an NPM)
- LPM can be used to simulate stable longitudinal networks

LPM simulated networks are compared to empirical networks and are shown to represent the network well.

<table>
<thead>
<tr>
<th>M</th>
<th>( \delta )</th>
<th>N</th>
<th>60000</th>
<th>t-val</th>
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<td>12.77754</td>
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</table>
Probability Background

- Consider a normal distribution with $\mu=0$ and $\sigma=1$.
- 95% of the time, observations are between $\pm 1.9597$
- When an observation occurs in the tail, we don’t believe it and think that something unusual might be going on.

\[
Z_t = \frac{x_t - \mu_0}{\sigma}
\]

Statistical Process Control

- Manufacturing processes are: stochastic, dependent, non-ergotic, complex, and involve human interaction.
- Shewhart (1927) X-bar Control Chart proposed to monitor change of any process
- Calculate $Z_t$ transform value for each time-period, $t$.
- Calculate a control limit, $L$, based on risk for false alarm.
- Chart Signals when $Z$ exceeds control limit, $L$. 
The Shewhart X-Bar Chart

- **Overview**
  - Fit normal distribution on “control period” (early observations) > assumed to represent the “normal state”
  - Signal change if a subsequent observation is outside confidence interval
- **Simple Example of technique**

![Graph showing normal distribution and signal change]

- **Parameters**
  - # observations used to fit distribution (the “normal” period)
  - False positive risk or decision interval
    - Trade-off between False positive risk & detection speed
- **Assumption**
  - Observations are normally distributed as independent random vars
    - Shewhart X-Bar chart used even when assumption is violated. However, false positive risk probability may be inaccurate
Statistical Process Control (cont.)

- Newer approaches detect change in fewer observations subject to the same rate of false positives.

- Scan Statistic (Fisher, 1934)

- Exponentially Weighted Moving Average (EWMA) (Roberts, 1959)
  - Good at detecting small changes in mean over time
  - Performs well on time series with closely spaced data samples

\[ w_t = \lambda \bar{x}_t + (1 - \lambda)w_{t-1} \quad \mu_0 \pm L\sigma_r \left( \frac{\lambda}{2 - \lambda} \left[ 1 - (1 - \lambda)^{2T} \right] \right)^{1/2} \]

- Cumulative-Sum (CUSUM) Control Chart (Page, 1961)
  - Good at detecting small changes in mean over time
  - Built-in change point detection
  - Two Charts (To Detect Increase and Decrease)

\[ C^+_i = \max \{0, Z_i - k + C^+_{i-1}\} \quad C^-_i = \max \{0, -Z_i - k + C^-_{i-1}\} \]

Cumulative Sum (CUMSUM)

- Cumulative-Sum Control Chart
  - Good at detecting small changes in mean over time
  - Built-in change point detection
- Calculate \( Z_t \) transform for each time-period, \( t \)

\[ Z_t = \frac{x_t - \mu_0}{\sigma} \]

- Two Charts (To Detect Increase and Decrease)

\[ C^+_i = \max \{0, Z_i - \frac{\delta}{2} + C^+_{i-1}\} \]

- Chart Signals when \( C^+ \) or \( C^- \) statistic exceeds decision interval

\[ C^-_i = \max \{0, -Z_i - \frac{\delta}{2} + C^-_{i-1}\} \]

Sensitivity in CUSUM due to discrete integration of error
Comparison of Change Detection Approaches

**Baseline Avg. Betweenness**

- **Over-Time Meas**
- **CUSUM Statistic**

### Simulation Time Period

<table>
<thead>
<tr>
<th>CUSUM Statistic</th>
<th>No Change</th>
<th>Change</th>
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<tr>
<td>Average Betweenness</td>
<td>9.32</td>
<td>8.24</td>
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<tr>
<td>Maximum Betweenness</td>
<td>14.36</td>
<td>14.72</td>
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<tr>
<td>Standard Deviation Betweenness</td>
<td>16.44</td>
<td>16.24</td>
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<tr>
<td>Average Closeness</td>
<td>10.68</td>
<td>9.08</td>
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<tr>
<td>Maximum Closeness</td>
<td>8.76</td>
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<tr>
<td>Standard Deviation Closeness</td>
<td>34.48</td>
<td>34.72</td>
</tr>
<tr>
<td>Average Eigenvector</td>
<td>31.28</td>
<td>31.28</td>
</tr>
<tr>
<td>Minimum Eigenvector</td>
<td>14.36</td>
<td>14.36</td>
</tr>
<tr>
<td>Maximum Eigenvector</td>
<td>5.24</td>
<td>5.40</td>
</tr>
<tr>
<td>Standard Dev Eigenvector</td>
<td>5.92</td>
<td>4.88</td>
</tr>
</tbody>
</table>

**CUSUM Statistic**

- **k = 0.5**
- **EWMA r = 0.1**
- **EWMA r = 0.2**
- **EWMA r = 0.3**
- **Scan Statistic**

Average Betweenness:
- CUSUM: 9.32
- EWMA r = 0.1: 8.24
- EWMA r = 0.2: 10.16
- EWMA r = 0.3: 11.52
- Scan: 6.76

Maximum Betweenness:
- CUSUM: 14.36
- EWMA r = 0.1: 14.72
- EWMA r = 0.2: 15.72
- EWMA r = 0.3: 17.08
- Scan: 13.24

Stan Dev Betweenness:
- CUSUM: 16.44
- EWMA r = 0.1: 16.24
- EWMA r = 0.2: 16.92
- EWMA r = 0.3: 18.52
- Scan: 15.24

Average Closeness:
- CUSUM: 10.68
- EWMA r = 0.1: 9.08
- EWMA r = 0.2: 13.60
- EWMA r = 0.3: 17.52
- Scan: 10.48

Maximum Closeness:
- CUSUM: 8.76
- EWMA r = 0.1: 6.00
- EWMA r = 0.2: 10.60
- EWMA r = 0.3: 37.96
- Scan: 8.64

Stan Deviation Closeness:
- CUSUM: 34.48
- EWMA r = 0.1: 34.72
- EWMA r = 0.2: 34.52
- EWMA r = 0.3: 35.68
- Scan: 27.08

Average Eigenvector:
- CUSUM: 31.28
- EWMA r = 0.1: 31.28
- EWMA r = 0.2: 31.28
- EWMA r = 0.3: 31.28
- Scan: 24.00

Minimum Eigenvector:
- CUSUM: 14.36
- EWMA r = 0.1: 14.36
- EWMA r = 0.2: 14.28
- EWMA r = 0.3: 15.56
- Scan: 14.88

Maximum Eigenvector:
- CUSUM: 5.24
- EWMA r = 0.1: 5.40
- EWMA r = 0.2: 5.80
- EWMA r = 0.3: 7.52
- Scan: 4.00

Std. Dev Eigenvector:
- CUSUM: 5.92
- EWMA r = 0.1: 4.88
- EWMA r = 0.2: 6.40
- EWMA r = 0.3: 6.96
- Scan: 3.64
Network Change Detection: Analysis of Real World Data

<table>
<thead>
<tr>
<th># Nodes</th>
<th>Time Periods</th>
<th>Method of Collection</th>
<th>Type of Relation</th>
<th>Design</th>
<th>Known Change</th>
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<tr>
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<td>Rating</td>
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<tr>
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<td>28</td>
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<td>Email</td>
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</table>

Network Change Detection: Analysis of Real World Data

Fraternity | Leavenworth 2007 | Leavenworth 2005 | Al-Qaeda
Winter A | Winter C | IkeNet 2 | IkeNet 3
Summary of Change Detection Across Data Sets

There is a trade-off between false positive and rapid detection

- Low risk of false alarm
  - Longer to detect change

- High risk of false alarm
  - Faster to detect change

False alarm risk set by dec interval

Faster Detection

Too little risk may prevent change detection all together

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<td>Leav 07</td>
<td>3</td>
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<td>5</td>
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<td>No F.A.</td>
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<td>May</td>
<td>Sept</td>
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<tr>
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<td>15</td>
<td>18</td>
<td>19</td>
<td>19</td>
<td>20</td>
</tr>
</tbody>
</table>
Change Detection Hands-On

- Based on Roger Federer 2010 data

• Analysis uses over-time changes in “measures” based on the network data
Select The Metanetwork

Custom Measure Selection
Use Search to Find Measure

Hint: Click Select Box at bottom to deselect all measures, then use search to find the ones you want.

Two measures selected to run

Add Measure – Agent Based Measure – select “Centrality, Total Degree”
Now Select Display

First step is to select type of variables to display – AGENT Level in this case

Then click on “Add Measure” to add a new plot line

Select Agent for Measure

Click on the “Click to choose ….” button and select second agent for analysis (Federer is always primary)
Add Another Agent Measure Plot

Click on Add a Measure button again to add another line

Change Detection Hands-On
The Shewhart X-Bar Chart

# of networks used to fit normal distribution

False positive probability

Change detected

Obama

Nadal
Change Detection Hands-On
CUMSUM Method

The $\delta$ parameter

Fast Fourier Transform (FFT)

- Goal: detect periodicity in over-time data
- Examples
  - Weekly periodicity in email data
  - Time of the day effects
- Fourier’s theorem
  - Any time signal can be represented by a sum of sinusoidal functions with different frequencies, amplitudes and phase shifts
- Fourier transform finds sinusoids that decompose a signal
  - Analogy: given a dish, find the ingredients
  - Sinusoids have the advantage that they are orthogonal
**Sinusoidal Function**

- A sinusoidal function \( y = A \sin(2\pi ft + \phi) \) has
  - \( A \) amplitude
  - \( f \) frequency \( (T = \frac{1}{f} \text{ is the period}) \)
  - \( \phi \) phase

\[ y = 3 \sin(2\pi(0.25)(t + 0.5)) \]

**Frequency Domain: The FFT**

- Fast Fourier Transform (FFT)
- Decompose time waveform into sum of sinewaves
FFT Example: Sum of Sinusoids

- Fast Fourier transform of sinusoidal function is a spike at the sinusoidal frequency
- Example $y = \sin(2\pi \cdot 0.25 \cdot t)$

FFT Example 2

- FFT finds periodicities that may be unclear in over-time signal

Hidden "recipe": over – time signal computed as $y(t) = 2\sin(2\pi \cdot 0.25 \cdot t) + 3\sin(2\pi \cdot 0.3 \cdot t + 0.2) + \text{noise}$
Fourier Analysis Example 3

- 24 cadets in a regimental chain of command agreed to have their email monitored to form a social network data set known as IkeNet3.
- The betweenness was calculated based on the e-mail communications observations over the first month in their duty positions.

![Graph showing betweenness over time]

Fourier Analysis – Example 3

- Fourier transform
- Symmetric around the midpoint
- 3 main components (in terms of magnitude)

That is why we typically only display from origin up to midpoint.
Digital Signal Processing

Time Sampled Input
sampling period $T_S$ → FFT → Sum of Sinusoids
(Amplitude, frequency, phase)

Digital Signal
Processing (DSP) → Sum of Sinusoids
(Amplitude, frequency, phase) → Inverse
FFT → Time Sampled Input
sampling period $T_S$

One Key Issue: Aliasing Sampling Limits

If we want DSP to work unambiguously, we choose to limit
maximum input frequency to $\leq \frac{1}{2T_S} = \frac{F_S}{2} = “$Nyquist Frequency” $
Digital Signal Processing

Fourier transform
Symmetric around the midpoint
3 main components (in terms of magnitude)

That is why we typically only display from origin up to midpoint

How do we use this?

One possible approach – big peaks are periodic “background”

Filtering

e.g., identify 3 main (high magnitude) components
keep them and remove FFT components
at all other frequencies
Inverse Fast Fourier Transform

This is the inverse Fourier transform of just the 3 selected components, which are then reconverted to time waveform.

There is a weekly, two week and three week cycle.

Anomaly Detection

The filtered pattern has been subtracted from the original. The red is what is left – the anomalies.
**FFT Example Hands-On**

- IkeNet data (IkeNet3-dynamic.xml)
  - Email exchange data among mid-career officers in a one-year graduate program at Columbia University
  - Granularity: day; Duration: month

**Over Time Measures**

Again View Measures Over Time
Selecting MetaNetwork

Selecting Measures
Selecting Measures to Plot

Density, Weighted & engent2agent

Run Shewhart to Detect Change

Run Change Detection Shewhart
10 points for basis
0.05 for false positives

Lots of apparent events
That could be Change
All upward changes
FFT Can Help with Periodic Patterns

FFT Power Spectral Density

“window effects”

weekly 4 day 3 day
FFT Power vs. Period

Adjust Power Threshold to just select “background” periodic variations to be removed.

NOTE – viewing the Dominant Periods is often more intuitive than PSD.

The long period values are artifact of finite length of data.

Original vs. FFT subtracted data

Remaining FFT components are converted back to time values (inverse FFT) and subtracted.

This removes some of the periodic patterns covering up the fundamental changes to be detected.
Shewhart on FFT subtracted data

Now use the Filtered Data
Run Change Detection
Shewhart
10 points for basis
0.05 for false positives
Only 1 + event and
2 – events
Open Question – is this better than original or not?

Shewhart on original data

Run Change Detection
Shewhart
10 points for basis
0.05 for false positives
Lots of apparent events
That could be Change
All upward changes
Fourier Analysis to Handle Periodicity

- Fourier analysis can effectively identify periodic trends in longitudinal network data.
- Identification of periodic trends can allow the analyst to aggregate relational data over the period to remove over-time dependence.
- The inverse Fourier transform of the significant period can be used to subtract off periodicity from longitudinal network data measures over time.
- Further exploration of wavelets may produce greater insights into network dynamics.

Scalability

- The change detection algorithm is linear, thus the time consuming part is calculating network measures.
- Networks with less than 20 nodes tend to have a higher variance in over time measures. When a link is added or removed, it affects (n-1)(n-2) triads.
- Requires at least 3 time periods: >2 to determine typical behavior and 1 to compare at each time point. In practice, 10+ network time points are preferred.
- No difference in number of required networks for each technique: CUSUM, EWMA, Scan Statistic, x-bar, etc.
- Wavelet/Fourier based approach needs many more time periods and complexity grows roughly as \#T(log(#T))
Limitations

- View findings on data with caution
- Slicing and dicing can distort conclusions
- Examine errors associated with technique through extensive simulations.
- Investigate more real world data sets.
- Investigate the degree to which network measures are correlated to understand the effects of compounding error.
- Investigate multi-dimensional network properties such as the cosine similarity between the triad census at different time periods.

Summary of Change Detection

- Rapid change detection may allow an analyst to get inside a decision cycle and shape network evolution.
- Simulation is important for modeling longitudinal network behavior.
- Isolating when networks change enables more focused study on the causes of evolution, shock, and mutation, which may lead to future predictive analysis.
- Statistical process control is a useful tool for understanding social behavior.
FFT Hands On Session

File – Fourier-Example-3.xml
Walk through analysis on screen and on your laptops

FFT Example Hands-On (1/4)

- IkeNet data (IkeNet3-dynamic.xml)
  - Email exchange data among mid-career officers in a one-year graduate program at Columbia University
  - Granularity: day
  - Duration: month
FFT Example Hands On (2/4)

FFT Example Hands On (3/4)
Fourier Analysis to Handle Periodicity

- Fourier analysis can effectively identify periodic trends in longitudinal network data.

- Identification of periodic trends can allow the analyst to aggregate relational data over the period to remove over-time dependence.

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Summary Results

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- Simulation is important for modeling longitudinal network behavior.

- Isolating when networks change enables more focused study on the causes of evolution, shock, and mutation, which may lead to future predictive analysis.

- Statistical process control is a useful tool for understanding social behavior.

Conclusions

- Change detection
  - Detect occurrence of shocks i.e. change due to reasons exogenous to the network

- Fourier analysis
  - Detect periodicity in over-time data